Indian Statistical Institute

Back Paper 2011-2012

B.Math Second Year

Analysis IV

Time : 3 Hours Date : 19.07.2012 Maximum Marks : 100 Instructor : Jaydeb Sarkar

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Answer all questions.

Q1. (15 marks) State and prove the Riemann's localization theorem for an integrable function f over $[-\pi, \pi]$.

Q2. (15 marks) Let f be a 2π-periodic and continuous function and the Fourier series of f converges uniformly. Prove that the Fourier series of f converges to f.

Q3. (10 + 10 marks) Find the Fourier series of $f(x) = |x|$ where $-\pi \leq x \leq \pi$. Also prove that

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}.$$

Q4. (10 marks) Let f be a continuously differentiable 2π-periodic function and suppose that $\int_{-\pi}^{\pi} f(x)\, dx = 0$. Prove that

$$\int_{-\pi}^{\pi} (f(x))^2\, dx \leq \int_{-\pi}^{\pi} (f'(x))^2\, dx.$$

Q5. (10 marks) Let $\sum_n c_n = s$. Prove that the series $\sum_n c_n$ is Cesaro summable and the Cesaro sum is s.

Q6. (15 marks) Prove that the Borel sets are measurable.

Q7. (15 marks) Let A be a measurable subset of $\mathbb{R}$ and $\epsilon > 0$ is given. Prove that there exists an open set $U \supseteq A$ such that $m(U \setminus A) < \epsilon$.